Data-Driven Output Regulation using Single-Gain Tuning Regulators

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Motivation: Renewable-dominated power systems

Source: Wikipedia (left) PNNL (right)

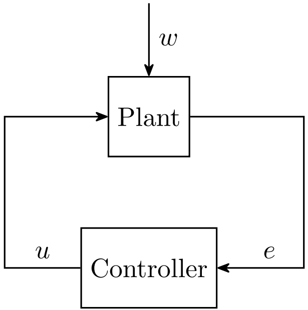
Desired controller features

- data-driven, minimal plant information requirement
- low-complexity, robust

Disturbance example

- gen./load imbalance causing frequency deviation from nominal value (60Hz)

Introduction: output regulation



- **Stable** LTI plant subject to exogenous disturbance w and control input u
- Dynamic controller with measurement error e as input

Objective: track reference and reject disturbance

A very brief history of output regulation

Linear systems

- Davison '72: feedforward measurable disturbance rejection
- Francis & Wonham '76: the internal model principle
- Davison '76: robust output regulation, multivariable tuning regulator for unknown MIMO systems
- Marino & Tomei '03, '14, '17: adaptive controller for SISO (discrete-time) unknown plant + known/unknown exosystem
- Wang & Davison & Davison '12, '13: discrete-time unknown plant + constant disturbance with input saturation

Nonlinear systems

- Isidori & Byrnes '90: nonlinear extension of Francis & Wonham '76 result
- Serrani, Isidori & Marconi '01: nonlinear system + unknown linear exosystem
- Huang & Chen '04, '05: robust nonlinear output regulation

and many more...

The Multivariable Tuning Regulator (Davison '76)

IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. AC-21, NO. 1, FEBRUARY 1976 35

Multivariable Tuning Regulators: The Feedforward and Robust Control of a General Servomechanism Problem

EDWARD J. DAVISON, MEMBER, IEEE

Problem setup

$$\begin{aligned} \dot{w} &= Sw \\ \dot{x} &= Ax + Bu + B_w w \\ e &= Cx + Du + D_w w \end{aligned} \qquad \begin{aligned} \mathrm{eig}\,(S) &\subset \overline{\mathbb{C}}^{+} \\ \mathrm{eig}\,(A) &\subset \mathbb{C}^{-} \end{aligned}$$

Available information

- Minimal polynomial of exosystem

$$\mu_S(s) = s(s^2 + \omega_1^2) \cdots (s^2 + \omega_\ell^2)$$

- Frequency response: for $k \in \{0, 1, \ldots, \ell\}$

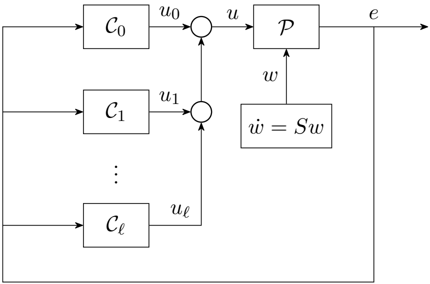
$$\hat{P}(\mathbf{j}\omega_k) = C(\mathbf{j}\omega_k I - A)^{-1} B + D$$

Architecture of Davison's Tuning Regulator

Controller structure

$$\mathcal{C}_k : \quad \begin{aligned} \dot{\eta}_k &= \Phi_k \eta_k + G_k \\ u_k &= -\epsilon_k F_k \eta_k \end{aligned}$$

- (Φ_k, G_k): internal model for the k-th disturbance frequency component
- F_k: control gain, computed based on (closed-loop) freq. response data
- ϵ_k: tuning parameters



Each F_k depends on re-identifying the frequency response of the previous closed-loop system $\{\mathcal{P}, \mathcal{C}_0, \ldots, \mathcal{C}_{k-1}\} \implies$ impractical if w is complex

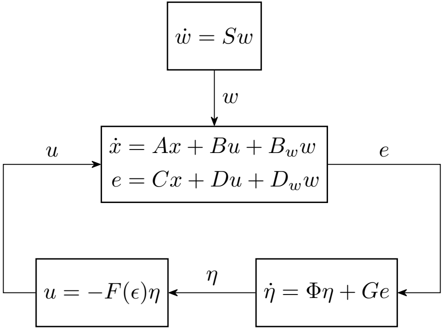
The Single-Gain Tuning Regulator

To address the drawbacks of Davison's design...

- controller gain computed with **open-loop** frequency-response data
- can be tuned with a **single** tuning parameter ϵ

$$\textbf{SGTR}: \quad \begin{aligned} \dot{\eta} &= \Phi\eta + Ge \\ u &= -F(\epsilon)\eta \end{aligned}$$

- (Φ, G): internal model stacked from (Φ_k, G_k)
- Gain F: cont. matrix-valued mapping, $\mathcal{O}(\epsilon)$ as $\epsilon \to 0^+$



The SGTR: Objective

Construct controller gain $F(\epsilon)$ to achieve CLS stability*

- Adjusting ϵ corresponds to (proportionally) changing the dominant CLS pole
- Definition: $\mathcal{A}(\epsilon)$ is **low-gain Hurwitz stable (LGHS)** if there exist constants $c, \epsilon^\star > 0$ such that for all $\epsilon \in [0, \epsilon^\star)$,

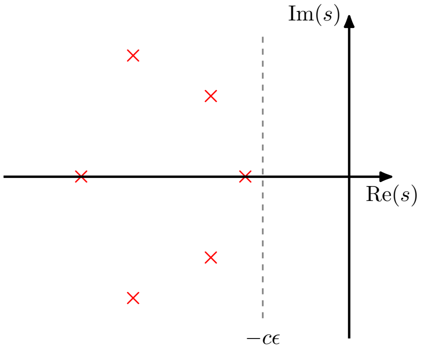
$$\max_{\lambda \in \mathrm{eig}(\mathcal{A}(\epsilon))} \mathrm{Re}[\lambda] \leq -c\epsilon$$

Objective

Show that the closed-loop matrix

$$\mathcal{A}(\epsilon) \triangleq \begin{bmatrix} A & -BF(\epsilon) \\ GC & \Phi - GDF(\epsilon) \end{bmatrix}$$

is low-gain Hurwitz stable



Certifying CLS LGHS: coordinate transformation

Coordinate transformation with Sylvester operator

- Define $x' \triangleq x - \Pi(\epsilon)\eta$, where $\Pi(\epsilon)$ solves the Sylvester equation

$$\mathrm{Syl}_{\Phi,A}(\Pi(\epsilon)) \triangleq \Pi(\epsilon)\Phi - A\Pi(\epsilon) = -BF(\epsilon)$$

Note: $\mathrm{Syl}_{\Phi,A}^{-1}$ always exists since $\mathrm{eig}\,(A) \subset \mathbb{C}^{-}$ and $\mathrm{eig}\,(\Phi) \subset \overline{\mathbb{C}}^{+}$

- Define $\mathcal{A}_{\mathrm{red}}(\epsilon) \triangleq \Phi - G\mathscr{L}(F(\epsilon))$, where

$$\mathscr{L}(F(\epsilon)) \triangleq -C\Pi(\epsilon) + DF(\epsilon) = C\mathrm{Syl}_{\Phi,A}^{-1}(BF(\epsilon)) + DF(\epsilon)$$

is the **steady-state loop gain (SSLG) operator**

Lemma (Reduction of closed-loop stability analysis)

$$\mathcal{A}_{\mathrm{red}}(\epsilon) \text{ is LGHS} \implies \tilde{A}(\epsilon) = \begin{bmatrix} A - \Pi(\epsilon)GC & \Pi(\epsilon)G\mathscr{L}(F(\epsilon)) \\ GC & \mathcal{A}_{\mathrm{red}}(\epsilon) \end{bmatrix} \text{ is LGHS}$$

Observations for controller gain design

- $\mathcal{A}_{\mathrm{red}}(\epsilon) \triangleq \Phi - G\mathscr{L}(F(\epsilon))$ looks like "$A - BK(\epsilon)$"
  Idea: Design $Z(\epsilon) = \mathscr{L}(F(\epsilon))$ using, e.g., pole placement

- Need to recover $F(\epsilon)$ for all stabilizing intermediary gain $Z(\epsilon)$
  Idea: The SSLG operator $\mathscr{L}$ needs to be surjective
- Fact: $\mathscr{L}$ is surjective if the non-resonance condition holds: $\forall \lambda \in \mathrm{eig}\,(S)$,

$$\mathrm{rank} \begin{bmatrix} A - \lambda I_n & B \\ C & D \end{bmatrix} = n + r$$

  Idea: If the non-resonance condition holds, then there exists a SGTR gain for the closed-loop system to be LGHS

- Problem: $\mathscr{L}(F(\epsilon)) = -C\Pi(\epsilon) + DF(\epsilon) = C\mathrm{Syl}_{\Phi,A}^{-1}(BF(\epsilon)) + DF(\epsilon)$ depends on the **unknown** (A, B, C, D)!

(Almost) model-free construction of $\mathscr{L}$

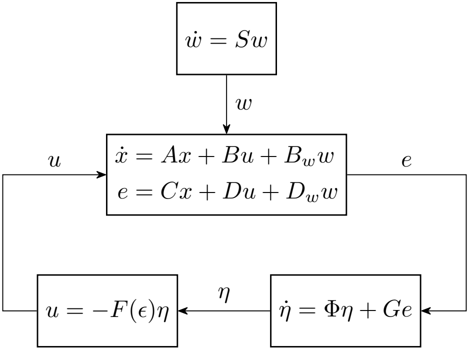
Theorem (Equivalent construction of SSLG operator)

The SSLG operator $\mathscr{L}$ can be alternatively constructed as

$$\mathscr{L}(F) = \hat{P}(0)F\boldsymbol{X}_0 + 2\sum_{k=1}^{\ell} \mathrm{Re}\{\hat{P}(\mathbf{j}\omega_k)F\boldsymbol{X}_k\}$$

- The matrices $\{\boldsymbol{X}_k\}$ for $k \in \{0, 1, \ldots, \ell\}$ are constructed from the eigen-decomposition of ϕ (recall $\Phi = \phi \otimes I_r$)
- Only depends on the open-loop freq. response and $\mu_S(s)$, not the plant info (A, B, C, D, B_w, D_w)
- A linear matrix equation in $F(\epsilon)$ if LHS is designed; can be reformulated to isolate $F(\epsilon)$ using Kronecker product identities

The SGTR: Summary



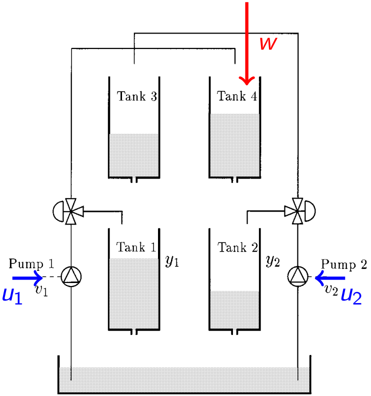
Design procedure

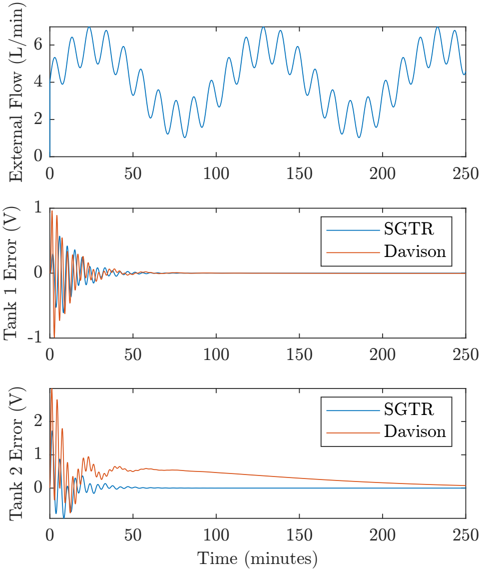
- Measure/estimate $\{\hat{P}(\mathbf{j}\omega_k)\}$, the freq. response data at the exosystem modes
- Design $Z(\epsilon) = \mathscr{L}(F(\epsilon))$ using, e.g., pole placement, $\mathcal{H}_\infty$ state-feedback
- Tune $\epsilon > 0$ and solve linear matrix equation in $F(\epsilon)$ for performance

Simulation & comparison against Davison's design

Four-tank process (Johansson TCST '00)

- Disturbance: external water flow into Tank 4

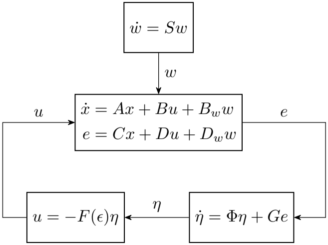




Conclusion

This paper

- Data-driven output regulator using frequency response data
- Easy to tune and implement
- Improvement of classical tuning regulator



Ongoing work

- Reduced-model based optimal & robust design
- Application: Power system load frequency control
- Discrete-time system extension: Focus on I/O data
- Incorporating feedforward + PID analogue
- Generalizing the SGTR to contractive systems

Questions

Appendix: Interpretation of the SSLG operator $\mathscr{L}$

- Small tuning parameter ϵ induces timescale separation
- Use **reduced model** to focus on the long term behavior; a candidate is

$$\begin{aligned} \dot{\eta} &= \mathcal{A}_{\mathrm{red}}(\epsilon)\eta + G\mathscr{L}_w w \\ e &= -\mathscr{L}(F(\epsilon))\eta + \mathscr{L}_w w \end{aligned}$$

$\mathscr{L}(F(\epsilon))$ is the steady-state model of the plant on the $\eta \to e$ channel