A Fixed-Point Algorithm for the AC Power Flow Problem

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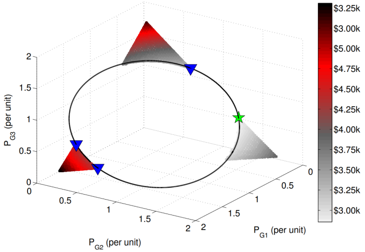
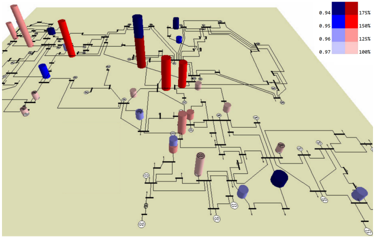
Background



Optimal power flow (Molzahn '17)



Contingency analysis (Sun & Overbye '04)

Fixed-point power flow algorithms in recent years

- Reformulate the standard $g(\xi) = \mathbb{0}$ form of power flow equations into an equivalent fixed-point form $\xi = f(\xi)$

Background

Advantages

- More robust against loading profile and initial condition variations
- Naturally lead to contraction-based algorithm convergence analysis

Contribution

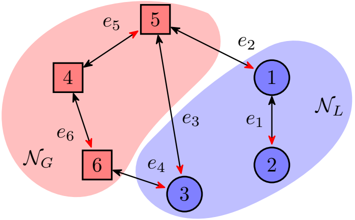
Extension of the lossless *Fixed-Point Power Flow* algorithm in the literature (JWSP '18) to include

- transmission line resistive losses
- phase-shifting transformers
- distributed slack buses in the network

Graph Modelling of an AC Transmission System

Weakly connected bidirected graph

- Each node $i \in \mathcal{N}$ models a bus
  - $\mathcal{N} = \mathcal{N}_L \cup \mathcal{N}_G$: the set of buses
  - $|\mathcal{N}_L| = n$ and $|\mathcal{N}_G| = m$
- Each edge $k \in \mathcal{E}$ models a branch
  - Π-model + transformer



Asymmetrically Weighted Incidence Matrix Γ

- Related to the the standard incidence matrix A
- Row $i \implies$ bus $i \in \mathcal{N}$; Column $k \implies$ branch $k \in \mathcal{E}$
- Γ_{ik}, Γ_{jk} represent the connection between buses i and j via the branch k
- Elements weighted by the conductance G and susceptance B (need separate Γ's for each)

The Power Flow Equations

The power flow equations with distributed slack bus

$$\bar{P}_i + \alpha_i P_{\mathrm{slack}} = \sum_{j=1}^{n+m} V_i V_j G_{ij} \cos(\theta_i - \theta_j) + V_i V_j B_{ij} \sin(\theta_i - \theta_j) \qquad i \in \mathcal{N}$$

$$Q_i = \sum_{j=1}^{n+m} V_i V_j G_{ij} \sin(\theta_i - \theta_j) - V_i V_j B_{ij} \cos(\theta_i - \theta_j) \qquad i \in \mathcal{N}_L$$

$$\Downarrow \quad \text{change of variable } \psi := \mathbf{sin}\left(A^{\mathsf{T}}\theta\right)$$

The equivalent fixed-point reformulation

$$R^{\mathsf{T}} \bar{P} = R^{\mathsf{T}} [V^{\circ}][g(v)][G_{ii}][V^{\circ}] g(v) + R^{\mathsf{T}} |\Gamma_G| [h(v)] \sqrt{\mathbb{1} - [\psi]\psi} + M_B [h(v)] \psi$$

$$Q_L = -[V_L^{\circ}][v][B_{ii}]_L [V_L^{\circ}] v + \Gamma_{G_L} [h(v)] \psi - |\Gamma_{B_L}| [h(v)] \sqrt{\mathbb{1} - [\psi]\psi}$$

The Proposed Fixed-Point Algorithm

Re-arranging the fixed-point reformulations...

$$v = \mathbb{1}_n - \frac{1}{4} S^{-1}[v]^{-1} \left(Q_L - \Gamma_{G_L}[h(v)]\psi - |\Gamma_{B_L}|[h(v)] \left(\mathbb{1}_{|\mathcal{E}|} - \eta \right) \right) \quad (1)$$

$$\psi = [h(v)]^{-1} \left(M_B^{\dagger} R^{\mathsf{T}} \left(\bar{P} - [V^{\circ}][g(v)][G_{ii}][V^{\circ}]g(v) - |\Gamma_G|[h(v)]\eta \right) + K x_c \right) \quad (2)$$

$$\mathbb{0}_{n_c} = C^{\mathsf{T}} \mathbf{arcsin}(\psi) \bmod 2\pi \quad (3)$$

Algorithm **The extended fixed-point power flow algorithm**

$v[0] \leftarrow V_L / V_L^{\circ}$, $\psi[0] \leftarrow \mathbf{sin}(A^{\mathsf{T}}\theta)$, $x_c[0] \leftarrow \mathbb{0}_{n_c}$, $k \leftarrow 0$

while power balance mismatch $> \epsilon$ AND $k <$ maximum iteration limit **do**

  update $v[k+1]$ using (1) with $v[k], \psi[k], x_c[k]$

  if $\mathcal{G}$ is not radial **then**

    update $x_c[k+1]$ using (3) with a Newton step

  update $\psi[k+1]$ using (2) with $v[k+1], \psi[k], x_c[k+1]$

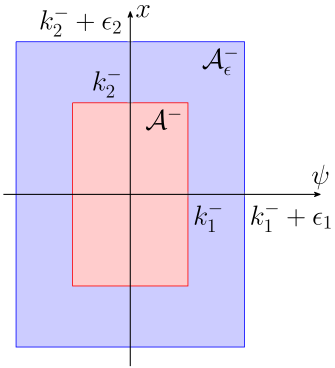
  $k \leftarrow k+1$

return $\psi[k+1]$, $v[k+1]$

Two-Bus Power Flow

The F_μ-invariant set $\mathcal{A}_\epsilon^-$

- $F_\mu : \mathbb{R}^2 \to \mathbb{R}^2$ characterizes the algorithm update rule
- $k_1^-, k_2^- \geq 0$ related to loading margins
- If μ is sufficiently small, then there exists $\epsilon_1, \epsilon_2 > 0$ such that $\mathcal{A}_\epsilon^- := \{\xi : |\psi| \leq k_1^- + \epsilon_1, |x| \leq k_2^- + \epsilon_2\}$ is F_μ-invariant

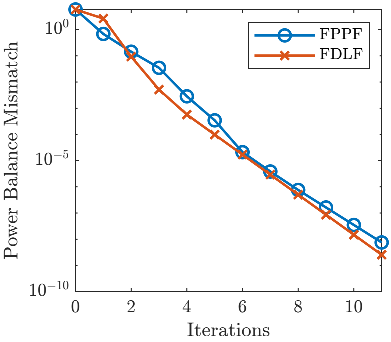


Main theoretical result: Two-bus power flow solvability

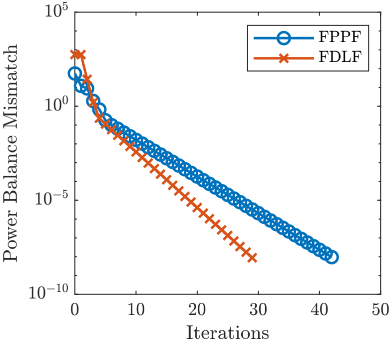
For sufficiently small μ, F_μ is a contraction on $\mathcal{A}_\epsilon^-$, so

- the unique high-voltage soln. is in $\mathcal{A}_\epsilon^-$
- FPPF always converges to this soln. from any $\xi_0 \in \mathcal{A}_\epsilon^-$

Numerical Results I: Convergence



118 bus system, base loading

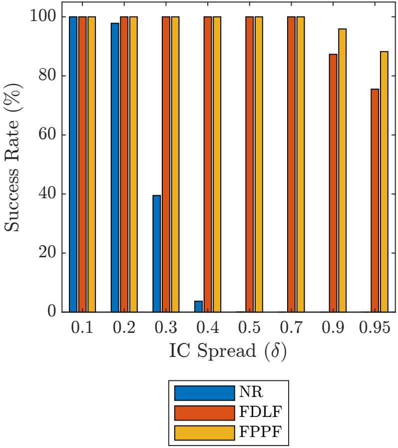


PEGASE 2869 bus system, high loading

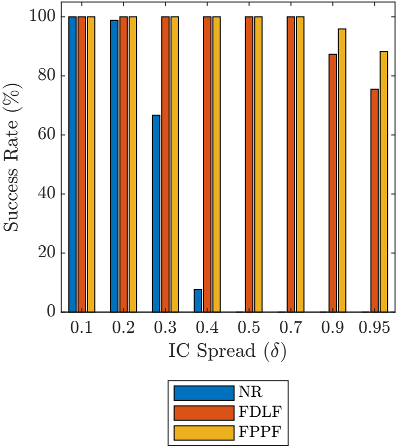
Conclusion: linear convergence, similar rate to the Fast-Decoupled Load Flow

Numerical Results II: Sensitivity to Initialization

- **Goal**: test algorithm success rate (%) under random initial load bus voltage magnitudes, generated uniformly from $[1-\delta, 1+\delta]$
- Higher success rate as δ increases $\implies$ more robust



30 bus system, high loading



118 bus system, high loading

The End

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